Normality Tests

The **NORMALITY TESTS** command performs hypothesis tests to examine whether or not the observations follow a normal distribution. The command performs following hypothesis tests - Kolmogorov-Smirnov (Lilliefors), Shapiro-Wilk W, D'Agostino-Pearson Skewness, Kurtosis and Omnibus K2 tests. Normal probability plot could be produced to graphically assess whether the sample comes from a normal distribution.

How To

- ✓ Run: **STATISTICS->BASIC STATISTICS->NORMALITY TESTS...**
- ✓ Select variables.
- ✓ Optionally, histogram with normal curve overlay could be plotted for each variable – use the **HISTOGRAM** option in the **ADVANCED OPTIONS**.

Results

Table with descriptive statistics and hypothesis tests results is produced for each variable.

SAMPLE SIZE, STANDARD DEVIATION, MEAN, MEDIAN, SKEWNESS, KURTOSIS, ALTERNATIVE SKEWNESS (FISHER'S), ALTERNATIVE KURTOSIS (FISHER'S) - see the **DESCRIPTIVE STATISTICS** procedure for more information.

Null hypothesis H_0: The data follow a normal distribution.
Alternative hypothesis H_1: The data do not follow a normal distribution.

Kolmogorov-Smirnov Test (with Lilliefors correction)

The Kolmogorov-Smirnov test (K-S test) compares sample data with a fitted normal distribution to decide if a sample comes from a population with a normal distribution. Test statistic is defined as

$$D = \max_{i=1..N} \left| CDF(x_i) - \frac{i-1}{N}, \frac{i}{N} - CDF(x_i) \right|,$$

where *CDF* is the normal cumulative distribution function.

When the CDF parameters are not known a priori, the test becomes conservative and loses power. The Lilliefors correction (K-S-L test) of the Kolmogorov-Smirnov test (Lilliefors 1967) estimates the mean and standard deviation of the CDF from the data. The correction uses different critical values and produces more powerful test.

P-VALUE calculation is based on the analytic approximation proposed by Dallal and Wilkinson (1986). If the **P-VALUE** is less than α (default value – 0.05), the null hypothesis (the distribution is normal) is rejected.

The Kolmogorov-Smirnov/**STEPHENS** test is a modification proposed by Stephens (1974). The p-value is based on published critical values and can range only from 0.01 to 0.15 and is provided *only for reference*.

Shapiro-Wilk W

The Shapiro-Wilk test, proposed by Shapiro in 1965, is considered the most reliable test for non-normality for small to medium sized samples by many authors. The test statistic is defined as:

$$W = \frac{\left(\sum_{i=1}^{N} a_i x_{(i)}\right)^2}{\sum_{i=1}^{N}(x_i - \bar{x})^2}.$$

Here $x_{(i)}$ is the ith sample value in ascending order, $\bar{x}$ is sample mean and a_i constants are defined as components of the vector $(a_1, \ldots, a_N) = \frac{m^T V^{-1}}{(m^T V^{-1} V^{-1} m)^{1/2}}$, where m are the expected values of the order statistics of independent and identically distributed *(i.i.d.)* random variables sampled from the *standard normal distribution* $(\mu = 0, \sigma = 1)$, and V is the covariance matrix of those statistics.

Anderson–Darling

The Anderson–Darling test checks if a given sample of data is drawn from a specific distribution. The test, proposed by Stephens in 1974, is a modified Kolmogorov-Smirnov test, but gives more weight to the tails of the distribution. The test statistic is defined as:

$$A^2 = -N - S,$$

where $S = \sum_{i=1}^{N} \frac{2i-1}{N}\left[\ln F(Y_i) + \ln(1 - F(Y_{N+1-i}))\right]$, F is the cumulative distribution function and Y_i are the ordered sample values. The better the distribution fits the data, the smaller is the value of the test statistic.

D'Agostino Tests

D'Agostino (1970) describes a normality tests based on the skewness b_1 and kurtosis b_2 coefficients. For the normal distribution, the theoretical value of skewness is zero, and the theoretical value of kurtosis is three.

D'Agostino Skewness

This test is developed to determine if the value of skewness b_1 is significantly different from zero.

The test statistic is defined as: $Z\left(\sqrt{b_1}\right) = \delta \ln\left(\frac{Y}{\alpha} + \sqrt{\frac{Y^2}{\alpha^2} + 1}\right)$, where the values Y, W, α, δ are defined in the following way:

$$Y = \sqrt{b_1 \frac{(n+1)(n+3)}{6(n-2)}}, \quad \beta_2 = \frac{3(n^2+27n-70)(n+1)(n+3)}{(n-2)(n+5)(n+7)(n+9)}$$

$$W^2 = -1 + \sqrt{2(\beta_2 - 1)}, \ \delta = 1/\sqrt{\ln W}, \ \alpha = \sqrt{2/(W^2 - 1)}$$

The test statistic *Z(b$_1$)* is approximately normally distributed under the null hypothesis of population normality. The null hypothesis of normality is rejected if the p-value is less than α level (0.05).

D'Agostino Kurtosis

This test is developed to determine if the value of kurtosis coefficient b_2 is significantly different from 3. The test statistic $Z(b_2)$ is approximately normally distributed $N(\mu(b_2), \mathrm{var}(b_2))$ under the null hypothesis of population normality.

$$Z(b_2) = \left(\left(1 - \frac{2}{9}A\right) - \sqrt[3]{\frac{1 - 2/A}{1 + x\sqrt{2/(A-4)}}} \right) / \sqrt{2/(9A)}$$

$$A = 6 + \frac{8}{\sqrt{\beta_1(b_2)}} \left[\frac{2}{\sqrt{\beta_1(b_2)}} + \sqrt{(1 + \frac{4}{\beta_1(b_2)})} \right], \; x = (b_2 - \mu(b_2))/\sqrt{\mathrm{var}(b_2)}$$

$$\sqrt{\beta_1(b_2)} = \frac{6(n^2 - 5n + 2)}{(n+7)(n+9)} \sqrt{\frac{6(n+3)(n+5)}{n(n-2)(n-3)}}$$

$$\mu(b_2) = \frac{3(n-1)}{n+1}, \; \mathrm{var}(b_2) = \frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)},$$

D'Agostino Omnibus

This test combines $\sqrt{b_1}$ and b_2 to produce an omnibus test of normality. The test statistic K^2 is approximately distributed as a chi-square with two degrees of freedom when the population is normally distributed. K^2 is defined as

$$K^2 = Z^2(b_1) + Z^2(b_2).$$

References

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