

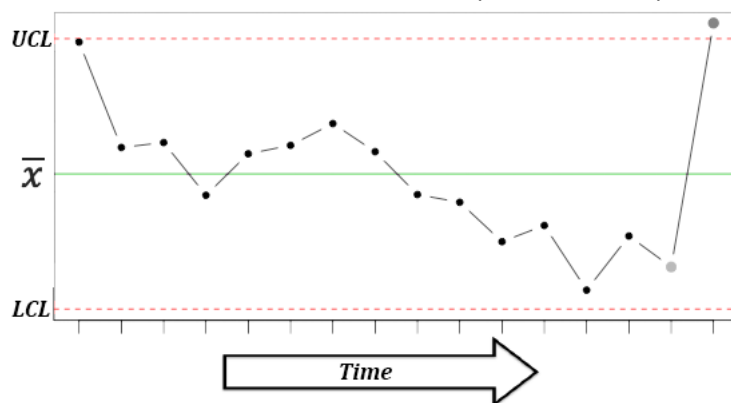
# Control Charts

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The control chart is a graph that represents the variability of a process variable over time. Control charts are used to determine whether a process is in a state of statistical control, to find the causes of changes in a process, and monitor process performance. Control charts are also known as Shewhart control charts, after W.A. Shewhart (1931) who first introduced the concept.

A control chart consists of:

- A center line, drawn as a green line at the mean value for the in-control process (*stable zone*).
- Upper and lower *control limits* (UCL and LCL) – red lines. These control limits are chosen so that almost all of the data points will fall within these limits as long as the process remains in-control. Control limits are set at a distance of three sigma  $3\sigma$  (standard deviation) above and below the mean centerline. Default distance ( $3\sigma$ ) can be overwritten using the “**CONTROL LIMITS AT THESE MULTIPLES OF STANDARD DEVIATIONS**” option.
- Data points representing a statistic for a subgroup (mean, range, proportion) or an attribute. A point outside the control limits indicates the presence of a special cause that deserves investigation.



There are two basic types of control charts – control charts for attributes and control charts for variables. Following types of control charts are available:

## *Individuals and moving range charts*

### *Control charts for subgroup averages*

- X-bar  $\bar{x}$  chart
- R chart
- s chart

### *Time weighted control charts*

- CUSUM chart

### *Control charts for attributes*

- P chart
- C chart
- U chart

## X-bar chart

X-bar  $\bar{x}$  chart is used to monitor the mean value of a process over time (*between-sample variability*). For each subgroup, the mean value is plotted.

- ✓ Run: **CHARTS->[CONTROL CHARTS] X-BAR AND R CHART OR X-BAR AND S CHART** command.
- ✓ Input variables: please see the **DATA LAYOUT FOR XBAR-R AND XBAR-S CHARTS** section below for more details.

## R chart

R chart is used to monitor the instantaneous process variability at a given time (*within-sample variability*). Standard deviation, approximated by the sample range is used as a measure of variability – for each subgroup, the range  $R_i = \max[x_i] - \min[x_i]$  is plotted. R charts are usually used when we have a constant and relatively small sample size (N=2...15). R chart can be produced for subgroups with a sample size up to 50.

- ✓ Run: **CHARTS->[CONTROL CHARTS] X-BAR AND R CHART** command.
- ✓ Input variables: please see the **Data layout for Xbar-R and Xbar-S charts** section below for more details.

### Methods

The default (*Rbar*) estimate for sigma is  $\sigma = \frac{1}{N} \sum_{i=1}^K R_i / d_2 (n_i)$ , where K is the number of subgroups,  $d_2$  is the unbiasing factor. Minimum variance linear unbiased estimate can be selected from the Advanced Options [v6.5+].

## S chart

S chart is similar to R chart, but the standard deviation is directly estimated. S charts are preferable when a sample size is a variable or moderately large (N>10).

- ✓ Run: **CHARTS->[CONTROL CHARTS] X-BAR AND S CHART** command.
- ✓ Input variables: please see the **Data layout for Xbar-R and Xbar-S charts** section below for more details.

### Methods

The default (*Sbar*) estimate for sigma is  $\sigma = \frac{1}{N} \sum_{i=1}^K S_i / c_4 (n_i)$ , where K is the number of subgroups,  $c_4$  is the unbiasing factor.

## Data layout for Xbar-R and Xbar-S charts

Use the **DATA LAYOUT** option to select how your data is organized.

### 1. SUBGROUPS ARE DEFINED BY VALUES OF THE GROUP VARIABLE

G	H
Length	Subgroup
146	Sample1
165	Sample1
165	Sample1
160	Sample1
154	Sample2
150	Sample2
168	Sample2
179	Sample2
179	Sample3
190	Sample3
153	Sample3
169	Sample3
174	Sample4
175	Sample4
180	Sample4
158	Sample4
192	Sample5
202	Sample5
181	Sample5
195	Sample5

Select the **LENGTH** variable as **MEASUREMENTS** and the **SUBGROUP** variable as **(DATA LAYOUT #1) GROUP**. This layout allows using subgroups with unequal sizes.

### 2. SINGLE COLUMN WITH MEASUREMENTS, FIXED SUBGROUP SIZE

G
Length
146
165
165
160
154
150
168
179
179
190
153
169
174
175
180
158
192
202
181
195

Select the **LENGTH** variable as **MEASUREMENTS** and enter **4** to the **(DATA LAYOUT #2) SUBGROUP SIZE** field.

### 3. SUBGROUPS ACROSS THE COLUMNS, EACH COLUMN REPRESENTS A SUBGROUP

A	B	C	D	E
Sample1	Sample2	Sample3	Sample4	Sample5
146	154	179	174	192
165	150	190	175	202
165	168	153	180	181
160	179	169	158	195

Select the **SAMPLE1–SAMPLE5** variables (columns A:E) as **MEASUREMENTS**. This layout allows using subgroups with unequal sizes.

### 4. SUBGROUPS ACROSS THE ROWS, EACH ROW REPRESENTS A SUBGROUP

J	K	L	M	N
Sample1	Case1	Case2	Case3	Case4
146	146	165	165	160
154	154	150	168	179
179	179	190	153	169
174	174	175	180	158
192	192	202	181	195

Select the **CASE1–CASE4** variables (columns K:N) as **MEASUREMENTS**.

*Workbook with samples is available through the Dataset link at the top of this page.*

## CUSUM chart

CUSUM chart (cumulative sum control chart) is used for change detection monitoring. While control charts for subgroup averages (X-bar, R and S charts) use information only from the last sample and can detect process changes greater than  $1.5\sigma$ , CUSUM chart, introduced by (Page, 1954), uses information given by the entire sequence of samples and thus can detect smaller shifts and maintain tight control over a process. CUSUM is one of the most powerful management tools available for the detection of trends and slight changes in data.

The cumulative sum of the deviations between each data point (a sample mean) and a reference value – target (T), is plotted. For the  $r^{th}$  sample the cumulative sum is defined as:

$C_r = \sum_{i=0}^r (x_i - T)$ , where T is the target. The choice of the T value depends on the application of the technique. Upper and lower control limits are computed as follows:

$$C_r^+ = \max[0, x_i - (T + K) + C_{r-1}^+], C_r^- = \max[0, (T - K) - x_i + C_{r-1}^-], C_0^+ = C_0^- = 0.$$

Allowance **K** is often chosen about halfway between the target **T** and the out-of-control value of the mean that we are interested in detecting.

- ✓ Run: **CHARTS->[CONTROL CHARTS] CUSUM CHART** command.
- ✓ Select a variable with group codes and a variable with measurements.
- ✓ Specify the target or check the “**AUTO SELECT TARGET**” option to use the grand mean as the target estimate. A grand mean is calculated as  $\frac{\sum_{subgroups} \sum_{i=1..Subgroup\ Size} X_i}{\sum_{subgroups} Subgroup\ Size}$ .
- ✓ Specify the decision interval **H** (*control limits*). Default value of H is 4.
- ✓ Specify the allowance **K** (also known as a slack value). Default value of K is 0.5, to detect one-sigma shifts in the mean.
- ✓ Specifies which subgroup to center the V-mask on. The subgroups have indices starting from 1. Leave the default value – zero, to use the last subgroup.

## P chart

P chart is used to monitor the proportion of defectives in a process. The *sample fraction nonconforming* (proportion of defectives) is defined as the ratio of the number of nonconforming units  $D$  to the sample size  $N$ :  $p = \frac{D}{N}$ . The random variable  $p$  follows binomial distribution. The mean of  $p$  is  $\mu_p = p$  and the variance of  $p$  is  $\sigma_p = \frac{p(1-p)}{N}$ . The control limits are defined as

$$LCL = Target\ value - 3\sigma_p, UCL = Target\ value + 3\sigma_p.$$

- ✓ Run: **CHARTS->[CONTROL CHARTS] P CHART** command.
- ✓ Select a variable with subgroups sample size and a variable with defectives count (*measurements*) for each subgroup.
- ✓ If the true fraction conforming  $p$  is known – specify the target value. When  $p$  is not known it is estimated with the grand mean.

## C chart

C chart is used to monitor the total number of nonconformities per unit  $c_i$ . Unlike the P chart, the C chart allows having more than one nonconformity per inspection unit, and requires a fixed sample size. The random variable  $c$  follows Poisson distribution.

Center line is defined as  $\bar{c} = \mu_c$ . It is estimated as the observed average number of nonconformities (*grand mean*), and control limits are defined as  $\bar{c} \pm k\sqrt{\bar{c}}$ , where  $k$  is set to 3 by default. If the lower control limit is negative, then there is no lower control limit.

- ✓ Run: **CHARTS->[CONTROL CHARTS] C CHART** command.
- ✓ Select a variable with measurements (number of nonconformities per unit).

## U chart

U chart is used to monitor the average number of nonconformities per unit  $u_i = c_i/N_i$ . With U chart we can have several inspection units in a sample. Center line is defined as  $\bar{u} = \mu_u$ .

- ✓ Run: **CHARTS->[CONTROL CHARTS] U CHART** command.
- ✓ Select a variable with subgroups sample size and a variable with defectives count (*measurements*) for each subgroup.

## Individuals and moving range I-MR charts

Individuals control chart (**I CHART**) is used for process monitoring based on individual observations. The center line represents an estimate of the process average (when unspecified, the average of all the observations, *grand mean*, is used), and points on the chart are the individual values. Control limits are defined as  $\bar{x} \pm k\overline{MR}/d_2$ , where  $\overline{MR}$  is an average of the moving ranges,  $d_2$  is the unbiasing factor and  $k$  is set to 3 by default. Individual charts are less sensitive (i.e., less powerful) than Xbar and R charts, and the assumption of normality is more critical (deviations from normality may lead to elevated rate of false alarms).

Moving range chart (**MR CHART**) monitors the between variation over time. Moving ranges represent point-to-point variation and are defined as the absolute differences between consecutive points. The number of points used to calculate a moving range is the moving range length or *span*. For the default span value of two,  $MR_i$  are defined as  $MR_i = |x_i - x_{i-1}|$ . Points on the chart are moving ranges  $MR_i$  and the center line is an average of the moving ranges  $\overline{MR}$ . Control limits are defined as  $\overline{MR} \pm kd_3\overline{MR}$  ( $k$  is set to 3 by default). Moving range chart may show cycles or pattern of runs induced by correlation of moving ranges (Montgomery, 2005) and thus should be treated cautiously.

- ✓ Run: **CHARTS->[CONTROL CHARTS] INDIVIDUALS AND MOVING RANGE** command.
- ✓ Select a variable with *measurements*.
- ✓ Optionally, change the method of estimating sigma.

## References

- Montgomery, D. (2005). Introduction to Statistical Quality Control. Hoboken, New Jersey: John Wiley & Sons, Inc.
- Oakland, J. (2007). Statistical Process Control, 6th edition. Butterworth-Heinemann.
- Page, E. S. (1954). "Continuous Inspection Scheme". Biometrika. 41 (1/2): 100–115.
- Shewhart, W.A. (1931). Economic Control of Quality of Manufactured Products. Van Nostrand, New York and MacMillan, London (501 pages).
- Wheeler, D. J. (2010). Individuals Charts Done Right and Wrong, Quality Digest.